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MULTI-MARKET MODELING OF AGRICULTURAL SUPPLY WHEN CROP LAND IS A QUASI-FIXED INPUT: A NOTE

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Abstract: Modeling of crop supply frequently adopts separate treatment of area and yield variables. The advantage of this approach is that it conveniently imposes the property of land being a quasi-fixed factor, at least on the aggregate. Given an agricultural land frontier, total supply of land may be fixed in the short run. Various crop multi-market models either ignore this property, thus foregoing the advantage of the area x yield formulation, or impose the aggregate land constraint in an ad hoc fashion. This note proposes a parsimonious area x yield framework of agricultural supply that is firmly rooted in optimization and requires minimal priors for calibration. The framework is designed for direct application in multi-commodity modeling of agricultural supply and demand.

Key words: supply response, area allocation, quasi-fixed factor

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1. **INTRODUCTION**

In discussions of medium to long term adequacy of food supply, *land* stands out among the various factors of production for agriculture. Malthus posits a race between “the means of subsistence” and population growth, with the former losing the battle of owing to scarce natural resources, especially land. The “check” on population has yet to arrive owing to amazing strides in raising yield over the past several decades. Nonetheless, Malthusian anxiety persists and has heightened, given the recent shift towards a regime of high and volatile world food prices, even as the world population passes a seven-billion milestone.

The area (scarce) and yield (growing) dichotomy is a useful conceptual distinction. While much intellectual energy has been expended on projecting the yield side of agricultural supply, relatively little effort has been spent understanding the area side. Yet the two elements are integral to understanding agricultural production. Conceptually, land may be treated as a quasi-fixed factor, while other inputs to farming are treated as flexible in quantity, e.g. labor, fertilizer, and even machinery (which can be purchased or rented). Current technology opens access to the farmer to raise yields (to levels unimaginable to Malthus) for as long as the flexible inputs can be applied to a fixed amount of land. The quantity of land can adjust but only beyond the short run.

The area x yield formulation is therefore an advantageous representation of agricultural supply. Furthermore, adapting this formulation in a multi-commodity setting reduces allocation of resources across crops to a land use problem. Mainstream modeling of multi-commodity agricultural supply does incorporate flexibility in area allocation, but unfortunately tends to gloss over the quasi-fixed nature of land. In this paper we outline a modeling framework integrating the area x yield formulation with flexible land use. The framework is highly tractable, firmly rooted in optimization, requires minimal priors for calibration, and is designed for applications to multi-commodity modeling of agricultural supply.

2. **SURVEY OF RELATED WORK**

Theory of area allocation

According to a review by Goddard (2010), Shumway, Pope, and Nash (1984) were first to investigate optimal area allocation under an aggregate area constraint, but inaccurately concluded that duality approaches offer little insight into this problem. Bewley, Young, and Colman (1987) eschew the simultaneous area-yield problem and directly posit instead a
multinomial logit model of crop area shares based on Theil (1969) which ensures satisfaction of non-negativity and adding-up conditions. The multinomial logit has been applied in subsequent literature, e.g. Khiem and Pingali (1995); Rosegrant, Kasrino, and Perez (1998).

Chambers and Just (1989) showed how to recast the area-yield problem within a dual optimization framework. Their choice model involved a two-stage decision-making procedure: in the first step the farmer selects the optimal level of inputs (and outputs) subject to an area constraint for each crop; in the second the farmer allocates the land area to the various crops. Coyle (1993) presented an econometric approach to implementing the optimal area allocation framework, but omitted discussion of yield determination.

Arnade and Kelch (2007) presented a yield and area allocation model based on duality, with land a quasi-fixed factor whether at the level of the farmer or of the industry. The sub-problem of optimizing output subject to an area constraint leads to shadow prices for land allocation; solving for these shadow prices jointly leads to an expression for the area elasticities.

Multi-market and general equilibrium models

A number of multi-market agricultural models currently in use apply the area-yield formulation for crop supply. In general however these models do not impose an aggregate land constraint for crop area. The AGLINK of FAO (Conforti and Londero, 2001) models crop area as a constant elasticity function of crop revenues per ha. The IMPACT of IFPRI also uses a constant elasticity formulation, with output prices as explanatory variables; a similar formulation is used in the China Agricultural Simulation Model or CAPSIM (Huang and Li, 2003).

General equilibrium models have also incorporated a special treatment for land. In the Global Trade in Agricultural Products (GTAP) model, two types of factors are distinguished, namely mobile and sluggish; the latter are characterized by an industry-specific rate of return, whereas returns per unit at the margin are identical for mobile factors (Hertel and Tsigas, 1997). Land appears as a sluggish factor; this essentially derives from an earlier CGE for the United States (Hertel and Tsigas, 1988). The production function is modeled directly, i.e. bypassing the area x yield formulation.

For the Philippines, the APSIM model (APPC, 2002) uses a variant of this approach by expressing crop area as a share of total. This indirectly incorporates the aggregate land constraint; however to satisfy the adding up restriction one of the crop categories is treated as a residual. Instead, deriving area allocation functions from profit maximization, as done in this
paper, directly incorporates the aggregate area constraint without need for *ad hoc* restrictions.

Alba and Briones (2010) identified in a general setting the minimal set of prior elasticities for inferring the Slutsky matrix of output supply and input demand functions. However when data are not sufficient or of poor quality, the optimization framework sketched in section 3 can aid in the calibration of the parameters for multi-output supply based on more austere priors. As shown by a parallels strategy proposed by Bouis (1996) for the demand side, greater parsimony comes at the cost of additional structure, and reliance on non-flexible yet still-plausible functional forms.

### 3. PRODUCTION WITH LAND ALLOCATION: BASIC MODEL

Under constant returns in all inputs, the production function can be expressed on average (per ha) basis. This allows us to represent optimal choice in two stages: in the first stage the farmer selects on per ha basis the optimal combination of inputs to produce output; in the second stage the farmer selects the optimal allocation of land across crops.

Let \( QS_i = A_i Y_i \) denote output, where \( QS \) denotes total crop output, \( A_i \) is hectarage planted/harvested, \( Y_i \) is output per ha, and \( i \) an index of crop category. Let \( X_{ij} \) denote average input \( j \) applied to a ha of land to produce crop \( i \). The corresponding input price is \( W_j \) while \( P_i \) is the output price and \( R_i \) the profit per ha. Farmers treat prices as given. The per hectare production function and profit are respectively as follows:
Equation (1) is in constant elasticity form; letting \( \sum_j \alpha_{ij} = \alpha_i \), we have \( \alpha_i < 1 \). Let value at the optimum be marked by an asterisk. Imposing the first-order condition for maximum profit, we arrive at a well-known result, which calibrates \( \alpha_{ij} \):

\[
W_j X_j^* = \alpha_{ij} P_i Y_i^*. 
\]

(3)

Substituting (3) in (2) and (1):

\[
R_i^* = P_i Y_i^* (1 - \alpha_i); 
\]

(4)

\[
Y_i^* = P_i^\frac{\alpha_i}{1-\alpha_i} \left[ \alpha_{0j} \prod_j (\alpha_{ij}/W_j)^{\alpha_{ij}} \right]^{\frac{1}{1-\alpha_i}}. 
\]

(5)

From the (natural) logarithm of (5) we obtain own-price elasticity of yield:

\[
\frac{\partial \log Y_i^*}{\partial \log P_i} = \frac{\alpha_i}{1-\alpha_i} > 0. 
\]

(6)

Equation (6) implies cross-price elasticities of yield are all zero. Taking the logarithm of (4), we obtain per ha profit elasticity with respect to price:

\[
\frac{\partial \log R_i^*}{\partial \log P_i} = \frac{1}{1-\alpha_i}. 
\]

(7)

Now we turn to land, for which the quantity under each crop is denoted \( A_i \). Let total area be \( A = \sum_i A_i \), assumed fixed; converting land use across crops is subject to a transformation function under constant elasticity:

\[
A = \left( \sum_i \beta_i A_i^\rho \right)^\frac{1}{\rho}. 
\]

(8)

The transformation function is linearly homogeneous, hence the “farmer” can be treated as a representative farmer; for convenience the entire crop output is assumed to be produced by this representative farmer. As shorthand, let \( \phi = \sum_i \beta_i A_i^\rho \), and let \( TR = \sum_i R_i^* A_i \) be total net revenue.

The optimal net revenue of a unit of land by crop category functions as an indirect price received by the farmer from allocating land. The Lagrangian for the constrained net revenue maximization problem is written as follows:
Let $k$ be an alternative index for crop category. From the first-order conditions of the Lagrangian we obtain (at optimal $A_k$):

$$ R_k^* A_k^* = \lambda \left( \frac{A_k}{\phi} \right) \beta_k A_k^\rho. $$

(9)

With some substitution and rearrangement we arrive at an expression to calibrate $\lambda$:

$$ TR^* = \lambda A. $$

(10)

That is, $\lambda$ serves as a indirect shadow price of land. Re-arranging (9), with change of index:

$$ A_i = \left( \frac{\lambda \beta_i}{R_i^*} \right)^{\frac{1}{1-\rho}} \frac{\rho}{A_i^{1-\rho}}. $$

(11)

The elasticity of transformation with respect to relative profit per ha is given by:

$$ \sigma = \frac{\partial \log (A_k / A_i)}{\partial \log (R_i^* / R_k^*)} = \frac{1}{1-\rho}. $$

(12)

Based on prior information on $\sigma$, $\rho$ can be calibrated based on (12). Under concavity of the transformation function, $\sigma < 0$; that is, an increase in the relative net revenue of $i$ reduces the relative area of $k$, or increases the relative area of $i$. Using (10) and (11), $\beta_i$ can be calibrated.

Lastly, the cross-price elasticity of relative output $Q_{S_k} / Q_{S_i}$ with respect to $P_i$ is straightforward and is of the expected sign:

$$ \frac{\partial \log (Q_{S_k} / Q_{S_i})}{\partial \log P_i} = \frac{\sigma - 1}{1-\sigma} < 0. $$

4. CONCLUDING REMARKS

To summarize: our basic model adopts a two-stage framework for optimizing output supply. In multi-market modeling, the baseline data set would contain complete information on prices and quantities, both of outputs and inputs, by commodity. This data set is enough to calibrate the unknown parameters, namely the output elasticity and constant term of the per ha production function. For the second stage we require only an estimate of the elasticity of
substitution between alternative uses of land, together with the baseline data set (and calibrated parameters from the first stage).

The basic approach can readily be extended. The first stage can accommodate more flexible functional forms, conditional on an explicit dual net revenue function. Meanwhile the second stage can be enriched by partitioning the set of crop categories into related crop types and nesting the transformation functions. For instance crops can be partitioned into temporary and perennial; land use allocation is described by a lower transformation function within these respective partitions. (Intuitively we expect higher transformation elasticity for temporary crops in absolute terms). The composite land use under each partition is then further combined within an upper transformation function (again intuitively, a lower absolute elasticity similar to that within perennials).

REFERENCES


